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First Semester M.Tech. Degree Examination, Dec.2013/Jan.2014

Applied Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find the binary form of the number 193. **(06 Marks)**

b. List some sources of errors. **(04 Marks)**

c. Find the truncation error in approximating the function, $y(x) = \log(1+x)$ by,

i) $\overline{y_1(x)} = x$

ii) $\overline{y_2(x)} = x - \frac{x^2}{2}$

iii) $\overline{y_3(x)} = x - \frac{x^2}{2} + \frac{x^3}{3}$

iv) $\overline{y_4(x)} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{3}$

over the range $0 \le x \le 1$ **(10 Marks)**

2 a. Solve
$$5x_1 + x_2 + x_3 + x_4 = 4$$
$$x_1 + 7x_2 + x_3 + x_4 = 12$$
$$x_1 + x_2 + 6x_3 + x_4 = -5$$
$$x_1 + x_2 + x_3 + 4x_4 = -6$$
by Gauss elimination method. **(06 Marks)**

b. Solve the following system of equations by using LU decomposition method:
$$x_1 + x_2 + 2x_3 = 4$$
$$2x_1 + 3x_2 + 4x_3 = 9$$
$$3x_1 + 4x_2 + 5x_3 = 11$$
(07 Marks)

c. Find the inverse of a matrix by the method of Gauss Jordan method,
$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & -1 & 1 \end{bmatrix}.$$
(07 Marks)

3 a. Use Jacobi's method to find the eigen values and eigen vectors of the matrix
$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}.$$
(10 Marks)

b. By employing the given's method reduce the matrix,
$$A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{bmatrix}$$
to tridiagonal form and hence find its dominant eigen value. **(10 Marks)**

4 The displacement of instrument subject to a random vibration test at different instant of time is found to be as follows:

Station (i)	1	2	3	4	5	6	7	8	9	10	11	12
Time (t sec)	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
Displacement Y_i (inches)	0.144	0.172	0.213	0.296	0.296	0.085	0.525	0.110	0.062	0.055	0.042	0.035

Determine the velocity $\left(\frac{dy}{dt}\right)$, acceleration $\left(\frac{d^2y}{dt^2}\right)$ and Jerk $\left(\frac{d^3y}{dt^3}\right)$ at t = 0.05, 0.20 and 0.60 second using suitable finite difference formula with a step size Δt of 0.05 second. **(20 Marks)**

5 a. Determine the value of integral $\int_0^1 \frac{dx}{1+x^2}$ using ,

i) Trapezoidal rule taking $h = \frac{1}{4}$.

ii) Simpson's $\frac{3}{8}$ rule taking $h = \frac{1}{6}$. **(10 Marks)**

b. Evaluate $\int_0^{\frac{1}{2}} \frac{dx}{\sin x}$ use Romberg's integration with step size $h = \frac{1}{16}$. **(10 Marks)**

6 a. Find the solution of the initial value problem $y' = (x+y)^{-1}$, y(0) = 1 taking h = 0.5. Find y(0.5) and y(1) using 4[th] order R-K method. **(10 Marks)**

b. Using the Adumn Bash forth method, determine y(0.4) given that $\frac{dy}{dx} = x + y$, y(0) = 1, y(0.1) = 1.1034, y(0.2) = 1.2428, y(0.3) = 1.3997. Apply the corrector formulae twice. **(10 Marks)**

7 a. The deflection of a beam is governed by the equation $\frac{d^4y}{dx^4} + 81y = \phi(x)$, where $\phi(x)$ is given by the table,

x	$\frac{1}{3}$	$\frac{2}{3}$	1
$\phi(x)$	81	162	243

and boundary condition $y(0) = y'(0) = y''(1) = y'''(1) = 0$. Evaluate the deflection at the pivotal points of the beam three sub-intervals. **(10 Marks)**

b. Determine approximate value of smallest characteristic value of λ for the problem $y'' + \lambda y = 0$, $y(0) = y(1) = 0$. **(10 Marks)**

8 a. Derive the equation governing the free vibration of a beam. **(10 Marks)**

b. Using Crank Nicholson's method, solve $\frac{\partial^2 u}{\partial x^2} = 16\frac{\partial u}{\partial t}$, 0 < x < 1, t > 0 given $u(x,0) = 0$, u(0, t) = 0, u(1, t) = 50t. Compute u for two steps in 't' direction taking $h = \frac{1}{4}$. **(10 Marks)**

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